

On the form of the power equation for modeling solar chimney power plant systems

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Abstract

Recently several mathematical models of a solar chimney power plant were derived, studied for a variety of boundary conditions, and compared against CFD calculations. The importance of these analyses is about the accuracy of the derived pressure drop and output power equation for solar chimney power plant systems (SCPPS). We examine the assumptions underlying the derivation and present reasons to believe that some of the derived equations, specifically the power equation in this model, may require a correction to be applicable in more realistic conditions. The analytical results are compared against the available experimental data from the Manzanares power plant.

Keywords:

Renewable energy, Solar chimney power plant, Mathematical analysis

Nomenclature

Variables

A	cross-sectional area
A_r	cross-sectional area of the collector ground
g	acceleration due to gravity
h	height
\dot{m}	air mass flow rate
p	pressure
\dot{W}	flow power

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q	heat transfer per unit mass
q''	heat flux
R	gas constant
T	temperature
ρ	density
u	velocity
c_p	specific heat capacity

Subscripts

i	inlet
o	outlet
c	collector
t	tower
$turb$	turbine
atm	atmospheric

Abbreviations

LHS	left hand side
RHS	right hand side

1. Introduction

Although the idea of the solar chimney power plant (SCPP) can be traced to the early 20th century, practical investigations on solar power plant systems started in the late 1970s, around the time of conception and construction of the first prototype in Manzanares, Spain. This solar power plant operated between 1982 and 1989 and the generated electric power was used in the local electric network [1, 2].

The basic SCPP concept (Fig. 1) demonstrated in that facility is fairly straightforward. Sunshine heats the air beneath a transparent roofed collector structure surrounding the central base of a tall chimney tower. The hot air produces an updraft flow in the chimney. The energy of this updraft flow is harvested with a turbine in the chimney, producing electricity. Experiments with the prototype proved the concept to be viable, and provided data used by a variety of later researchers. A major motivation for subsequent studies lay in the need for reliable modeling of the operation of a

large-scale power plant. The Manzanares prototype had a 200 m tall chimney and a 40,000 m² collector area. Proposals for economically competitive SCPP facilities usually feature chimneys on the scale of 1 km and collectors with multiple square kilometer areas.

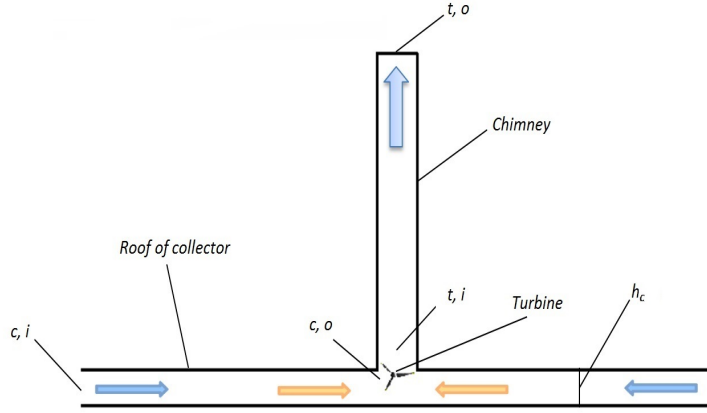


Figure 1: Schematic of SCPP with the applied variables and subscripts in the present analysis

Padki and Sherif [3] used the results from the Manzanares prototype to extrapolate the data to large scale models for SCPP. In 1991, Yan et al. [4] developed an SCPP model using a practical correlation. They introduced equations including air velocity, air flow rate, output power, and thermofluid efficiency. Von Backström and Fluri conducted a numerical study to determine the optimum ratio of pressure drop of the turbine as a fraction of the available pressure difference required to achieve the maximum power [5]. They noted that this ratio might lead to overestimating the flow passage in the plant and also designing a turbine without a sufficient stall margin. In other recent works, the SCPP concept involving an inflatable tower was examined, with all parts of the power plant modeled numerically [6, 7, 8]. A small-scale inflatable tower was fabricated for validation of these results, and code calibration was performed using the newly available experimental data [9, 10].

To find the maximum power, different atmospheric pressure and temperature boundary conditions were applied for various tower heights and atmospheric lapse rates [11]. Theoretical analysis to study the effect of pressure

drop in the SSCP turbine was performed by Koonsrisuk et al. [12]. The optimal pressure drop ratio was found numerically and analytically by Gue et al., around 0.9 for the Manzanares prototype. This investigation can be applied as an initial estimation for various SSCP turbines [13]. Tayebi et al. modeled and simulated the SSCP with a curved conjunction between tower and collector for different Rayleigh numbers [14].

Earlier modeling efforts [7] showed a keen sensitivity of the predictions of SSCP output to boundary conditions, in particular, pressure. Numerical simulations require careful validation and verification, and for that, analytical models are indispensable. A theoretical model was recently developed [15] to model the combined performance of the solar collector, chimney, and turbine. Here we will examine some of the assumptions and derivations in this model and present an alternative formulation for the energy equation.

2. Derivation of equations

2.1. Collector

To derive the equations, we start from the collector. It is assumed that the flow through the collector is one-dimensional, steady-state, and compressible. Let us disregard the friction and assume the total heat from the solar irradiation is absorbed within the air filling the collector. For this one dimensional axisymmetric compressible flow analysis, the mass conservation satisfies:

$$\frac{dA}{A} + \frac{d\rho}{\rho} + \frac{du}{u} = 0 \quad (\text{Continuity}) \quad (1)$$

Here A is the cross-sectional area of the collector that air goes through – $A = 2\pi r h_c$ and $dA = 2\pi r dh_c$.

Momentum equation is as follows [16]:

The momentum equation can be obtained from:

$$\frac{dp}{dr} + \frac{\rho u du}{dr} = 0 \quad (2)$$

$$dp + \rho u du = 0 \quad (\text{Momentum}) \quad (3)$$

The energy equation and the equation of states are

$$\frac{\rho u de}{dr} + \frac{p du}{dr} + \frac{k d^2 T}{dx^2} = 0 \quad (4)$$

$$p = p(\rho, T), e = e(\rho, T) \quad (5)$$

The last two equations, (5), represent the thermal and caloric equations of state [17]. Consider the energy balance equation and the equation of state as follows:

$$c_p dT - dq + u du = 0 \quad (\text{Energy}) \quad (6)$$

$$\frac{dp}{p} - \left(\frac{d\rho}{\rho} + \frac{dT}{T} \right) = 0 \quad (\text{State}) \quad (7)$$

To find dp we can apply Eq. (3) and substitute du/u from the continuity equation, Eq. (1).

$$-\frac{dp}{\rho u^2} = \frac{du}{u} = -\frac{d\rho}{\rho} - \frac{dA}{A} \quad (8)$$

$$dp = \rho u^2 \left(\frac{d\rho}{\rho} + \frac{dA}{A} \right) \quad (9)$$

From the equation of state we can find $d\rho/\rho$ and substitute in Eq. (9),

$$\frac{d\rho}{\rho} = \frac{dp}{p} - \frac{dT}{T} \quad (10)$$

$$dp = \rho u^2 \left(\frac{dp}{p} - \frac{dT}{T} + \frac{dA}{A} \right) \quad (11)$$

We can rewrite Eq. (11) as a function of T, A, u, p, ρ , where $\dot{m} = \rho A u$:

$$dp = \frac{\rho^2 u^2 A^2}{\rho} \left(\frac{dp}{A^2 p} - \frac{dT}{A^2 T} + \frac{dA}{A^3} \right) \quad (12)$$

Also by substitution dT from the energy equation on the base of dq, c_p and u , we obtain

$$dp = \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{dq - u du}{A^2 T c_p} + \frac{dp}{A^2 p} \right) \quad (13)$$

For consistency with previous analyses, let us rewrite dq on the basis of heat flux per mass flow rate— $dq = q'' dA_r / \dot{m}$ where q has the units of J/kg . Here $A_r = \pi r^2$, therefore $dA_r = 2\pi r dr$. Note that $A = 2\pi r h_c$, where h_c is

the collector height (roof height) that was assumed to be proportional to $r - h_c = ar$, where a is a constant. By substituting A_r , dq and A in the second term on the RHS, we obtain

$$dp = \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{q''(2\pi r)dr}{\dot{m}(2\pi r^2 a)^2 T c_p} + \frac{udu}{A^2 c_p T} + \frac{dp}{A^2 p} \right) \quad (14)$$

We can rewrite equation (14) and substitute udu of the third term on the RHS by applying momentum equation (3), $udu = -dp/\rho$.

$$dp = \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{q'' dr}{2\pi \dot{m} r^3 a^2 c_p T} - \frac{dp}{A^2 \rho c_p T} + \frac{dp}{A^2 p} \right) \quad (15)$$

Then we can substitute p from the equation of state, $p = \rho RT$ and rewrite the above equation to find dp on the LHS.

$$dp = \frac{\dot{m}^2}{\rho} \left[\frac{dA}{A^3} - \frac{q'' dr}{2\pi \dot{m} r^3 a^2 c_p T} \right] \left[1 - \frac{\dot{m}^2}{A^2 \rho^2 T} \left(\frac{1}{R} - \frac{1}{c_p} \right) \right]^{-1} \quad (16)$$

Note that

$$\dot{m} = \rho A u,$$

So we can rewrite (16) as

$$dp = \frac{\dot{m}^2}{\rho} \left[\frac{dA}{A^3} - \frac{q'' dr}{2\pi \dot{m} r^3 a^2 c_p T} \right] \left[1 - \frac{u^2}{T} \left(\frac{1}{R} - \frac{1}{c_p} \right) \right]^{-1} \quad (17)$$

Equations (16) and (17) are the exact solutions for dp for the one-dimensional frictionless analysis of the collector. Since our fluid is air we can estimate c_p and rewrite Eq. (17).

$$dp \simeq \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{q'' dr}{2\pi \dot{m} r^3 a^2 c_p T} \right) \left(1 - \frac{2.494 u^2}{T} \right)^{-1} \quad (18)$$

On the basis of mass flow rate, Eq. (18) can be written as:

$$dp \simeq \frac{\dot{m}^2}{\rho} \left(\frac{dA}{A^3} - \frac{q'' dr}{2\pi \dot{m} r^3 a^2 c_p T} \right) \left(1 - \frac{2.494 \dot{m}^2}{T \rho^2 A^2} \right)^{-1} \quad (19)$$

The third term of the RHS of Eq. (19) was ignored [15] which can be correct when density is constant. c_p , q'' and T are considered approximately constant as well. Therefore by integrating between the inlet and outlet of

the collector without the last term of the RHS, pressure difference can be derived.

$$\int_{c,o}^{c,i} dp \simeq \int_{c,o}^{c,i} \left(\frac{\dot{m}^2 dA}{\rho A^3} - \frac{\dot{m} q'' dr}{2\pi r^3 a^2 \rho c_p T} \right) \quad (20)$$

$$p_{c,i} - p_{c,o} \simeq \left[\frac{\dot{m}^2}{2\rho} \left(\frac{1}{A_{c,o}^2} - \frac{1}{A_{c,i}^2} \right) - \frac{q'' \dot{m}}{4\pi a^2 \rho c_p T} \left(\frac{1}{r_{c,o}^2} - \frac{1}{r_{c,i}^2} \right) \right] \quad (21)$$

2.2. Tower

The air flow in the tower(chimney) is considered as an adiabatic frictionless flow. The conservation equations for the one-dimensional steady state flow in variable-area tower are as follows:

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0 \quad (\text{Continuity}) \quad (22)$$

$$\frac{dP}{\rho} + g dz + u du = 0 \quad (\text{Momentum}) \quad (23)$$

$$c_p dT + u du + g dz = 0 \quad (\text{Energy}) \quad (24)$$

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0 \quad (\text{State}) \quad (25)$$

By following the same trend to find dp we get

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} + \rho u^2 \left(\frac{dp}{p} - \frac{dT}{T} \right) \right] \quad (26)$$

By applying the energy equation and substitution $dT = (-g dz - u du)/c_p$, we can rewrite the above equation as

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} + \rho u^2 \left(\frac{dp}{p} + \frac{g dz + u du}{c_p T} \right) \right] \quad (27)$$

Here $dp = -\rho(u du + g dz)$, then we get

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} + \rho u^2 \left(\frac{dp}{p} - \frac{dp}{\rho c_p T} \right) \right] \quad (28)$$

Above equation can be solved for dp ,

$$dp = \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} \right] \left[1 - \frac{u^2}{T} \left(\frac{1}{R} - \frac{1}{c_p} \right) \right]^{-1} \quad (29)$$

Also by considering the material properties of air the same way we did for the collector part,

$$dp \simeq \left[-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} \right] \left[1 - \frac{2.494u^2}{T} \right]^{-1} \quad (30)$$

The above equation is the exact closed form solution of dp at any point on the base of variable ρ , T and q'' . We ignore the last term on the RHS by having a constant density [15] and integrate between the inlet and outlet tower area to find the pressure difference of the chimney as,

$$\int_{t,0}^{t,i} dp \simeq \int_{t,0}^{t,i} \left(-\rho g dz + \frac{\dot{m}^2 dA}{\rho A^3} \right) \quad (31)$$

$$p_{t,i} \simeq p_{t,o} + \rho g h_t + \frac{\dot{m}^2}{2\rho} \left(\frac{1}{A_{t,o}^2} - \frac{1}{A_{t,i}^2} \right) \quad (32)$$

To calculate the output power, we can define the power on the basis of the pressure difference at the turbine – where it is normally utilized at the outlet of the collector and inlet of the tower.

$$\dot{W} \simeq \frac{\dot{m}(p_{c,o} - p_{t,i})}{\rho_{turb}} \quad (33)$$

Let $\rho_{turb} = (\rho_{c,o} + \rho_{t,i})/2$ and substitute equations $p_{c,o}$ and $p_{t,i}$ from (21) and (32). Hence for the flow power by assuming $p_{c,i} = p_{t,o} + \rho g h_t$, we have

$$\dot{W} = \frac{\dot{m}}{(\rho_{c,o} + \rho_{t,i})/2} \left[\frac{-\dot{m}^2}{2\rho} \left(\frac{1}{A_{c,o}^2} - \frac{1}{A_{c,i}^2} \right) + \frac{q'' \dot{m}}{4\pi a^2 \rho c_p T} \left(\frac{1}{r_{c,o}^2} - \frac{1}{r_{c,i}^2} \right) - \frac{\dot{m}^2}{2\rho} \left(\frac{1}{A_{t,o}^2} - \frac{1}{A_{t,i}^2} \right) \right] \quad (34)$$

For area the following equations are used, where b and c are arbitrary positive real constants.

$$A_{c,i}^2 = b A_{c,o}^2, \quad A_{t,o}^2 = c A_{t,i}^2 \quad (35)$$

The simplified form of equation (34) by applying the area correlations is,

$$\dot{W} \simeq \frac{\dot{m}}{(\rho_{c,o} + \rho_{t,i})/2} \left[\frac{-\dot{m}^2}{2\rho} \left(\frac{b-1}{bA_{c,o}^2} + \frac{1-c}{cA_{t,i}^2} \right) + \frac{q'' \dot{m}}{4\pi a^2 \rho c_p T} \left(\frac{1}{r_{c,o}^2} - \frac{1}{r_{c,i}^2} \right) \right] \quad (36)$$

Assume ρ is constant before and after turbine, therefore

$$\rho_{turb} = \rho_{c,o} = \rho_{t,i} = \rho$$

Koonsrisuk et al. derived an equation in which the second term was neglected in comparison with the first term on the RHS of Eq. (36). However, Eq. (37) shows the derived power equation by them at the end is likely to exceed the expected amount by a factor of two.

$$\dot{W} \simeq \frac{-\dot{m}^3}{2\rho^2} \left(\frac{1-c}{cA_{t,i}^2} + \frac{b-1}{bA_{c,o}^2} \right) \quad (37)$$

To evaluate the derived analytical solution for the output power of SCCP, the available experimental data from Manzanares prototype was applied and extracted. The measured updraft velocity for 25 hours Manzanares power plant operation is imposed to the analytical solution and the analytical power compared against the experimental outpower from the turbine (Fig. 2).

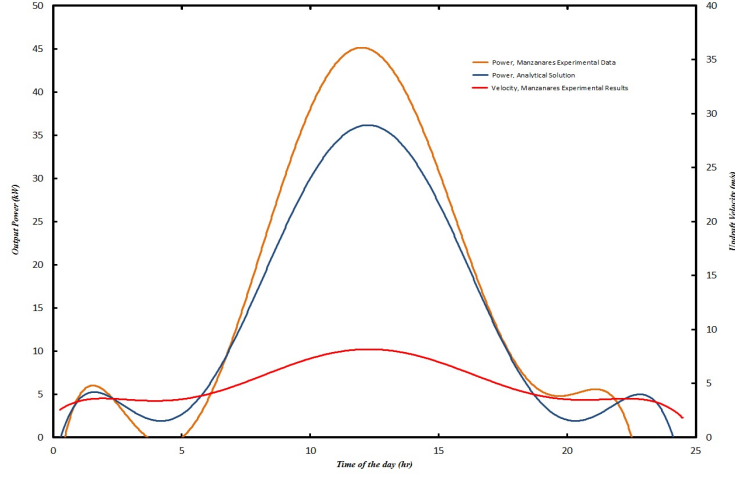


Figure 2: Analytical power results against measurement from Manzanares: updraft velocity and power output for a typical day

3. Conclusion

We presented considerations supporting our contention that a known analytical model predicting a solar chimney power plant performance may have a power equation which is off by a factor of two. Careful derivation of the models is very important, especially for the specific area of interest related to solar-chimney power plants, where numerical model scalability is a key issue, and few experimental results are available for validation. During the verification and validation process, the modeler must ask two questions: "Am I modeling the physics correctly?" and "Am I modeling the correct physics?" Comparison with analytical models is important for answering both of these questions, and the only way to have them well-posed is to have correct physics in the analytics.

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